

A blue robot with a screen on its head and a key in its right hand is pointing with its left hand to a graph. The graph shows a red parabola opening upwards, labeled $y_1 = f(x)$ at its top right. A green line is tangent to the parabola at a point marked with a green dot. The origin is labeled '0'. Another blue parabola, labeled $y_2 = f(x)$ at its bottom, is also shown, passing through the origin. The axes are labeled 'x' and 'y'.

EXAMPLE 1: Solve the system: $\begin{aligned} 5x - 2y &= 20 \\ 3x + 7y &= -29 \end{aligned}$

Ch 73 –Systems of Equations, Elimination

$$\begin{array}{rcl} 5x - 2y = 20 & \xrightarrow{\text{times } 3} & 15x - 6y = 60 \\ 3x + 7y = -29 & \xrightarrow{\text{times } -5} & -15x - 35y = 145 \end{array}$$

Add the equations: $0x - 41y = 205$

The x 's are gone!

Divide by -41 : $\frac{-41y}{-41} = \frac{205}{-41} \Rightarrow \underline{y = -5}$

Now that we have the value of y , we can substitute its value of -5 into either of the two original equations to find the value of x .

Using the first equation:

$$\begin{aligned} 5x - 2(-5) &= 20 \\ \Rightarrow 5x + 10 &= 20 \\ \Rightarrow 5x &= 10 \\ \Rightarrow \underline{x = 2} \end{aligned}$$

Therefore, the final solution to the system of equations is

$$x = 2 \text{ \& } y = -5$$

EXAMPLE 2: Solve the system:
$$\begin{aligned} 12a + 7b &= 9 \\ -18a - 5b &= 3 \end{aligned}$$

Solution: Notice how the numbers we choose to multiply each equation by will accomplish our goal: They make the α 's disappear when the equations are added.

$$\begin{array}{rcl} 12a + 7b = 9 & \xrightarrow{\text{times 3}} & 36a + 21b = 27 \\ -18a - 5b = 3 & \xrightarrow{\text{times 2}} & -36a - 10b = 6 \end{array}$$

Add the equations: $0a + 11b = 33$

Divide by 11: $b = 3$

Placing this value of b into the first equation gives us

$$12a + 7(\mathbf{3}) = 9 \Rightarrow 12a + 21 = 9 \Rightarrow 12a = -12 \Rightarrow \underline{a = -1}$$

We now have our complete solution to the system of equations:

$a = -1 \text{ \& } b = 3$

Let's use this example to learn how to check our solution. The main theme is this: The values of a and b must work in both of the original equations in order to constitute a valid solution.

1st equation:

$$12a + 7b = 9$$

$$12(-1) + 7(\mathbf{3}) \stackrel{?}{=} 9$$

$$-12 + 21 \stackrel{?}{=} 9$$

$$9 = 9 \quad \checkmark$$

2nd equation:

$$-18a - 5b = 3$$

$$-18(-1) - 5(\mathbf{3}) \stackrel{?}{=} 3$$

$$18 - 15 \stackrel{?}{=} 3$$

$$3 = 3 \quad \checkmark$$

Our conclusion is that the values $a = -1$ and $b = 3$ work perfectly. The solution can also be written as the ordered pair $(-1, 3)$.

Homework

1. Solve each system using the Elimination Method, and be sure you practice checking your solution (your pair of numbers) in both of the original equations:

a.
$$\begin{aligned} 2x + y &= 5 \\ -2x + 7y &= 19 \end{aligned}$$

b.
$$\begin{aligned} 5a - 3b &= 5 \\ 10a + 4b &= -40 \end{aligned}$$

c.
$$\begin{aligned} -2u - 3v &= -16 \\ -7u + 8v &= -56 \end{aligned}$$

d.
$$\begin{aligned} 7x + 12y &= -24 \\ 6x - 7y &= 14 \end{aligned}$$

e.
$$\begin{aligned} 3m - 2n &= 34 \\ -6m + n &= -62 \end{aligned}$$

f.
$$\begin{aligned} -3s - 3t &= -24 \\ 10s + 8t &= 64 \end{aligned}$$

g.
$$\begin{aligned} 2c - 3d &= 13 \\ 5c + 6d &= -8 \end{aligned}$$

h.
$$\begin{aligned} -5w - 4x &= -20 \\ 20w + 3x &= 15 \end{aligned}$$

i.
$$\begin{aligned} -5x - 4n &= -8 \\ 11x + 6n &= -2 \end{aligned}$$

j.
$$\begin{aligned} 2w - 4a &= 6 \\ -3w + 9a &= -12 \end{aligned}$$

k.
$$\begin{aligned} 2n - 3y &= -2 \\ 8n - 11y &= -2 \end{aligned}$$

l.
$$\begin{aligned} 4c + 9y &= 4 \\ -5c - 11y &= 12 \end{aligned}$$

m.
$$\begin{aligned} 5g - 2h &= -6 \\ 4g + 2h &= 3 \end{aligned}$$

n.
$$\begin{aligned} -4w + 3h &= -1 \\ -3w + 4h &= 5 \end{aligned}$$

o.
$$\begin{aligned} -3w + 4m &= 6 \\ -3w - m &= 1 \end{aligned}$$

p.
$$\begin{aligned} 3a + 3q &= 1 \\ -5a + 5q &= 6 \end{aligned}$$

Solutions

1. a. $x = 1, y = 3$

Complete Check:

$$2x + y = 5$$

$$2(1) + 3 = 5$$

$$2 + 3 = 5$$

$$5 = 5 \quad \checkmark$$

$$-2x + 7y = 19$$

$$-2(1) + 7(3) = 19$$

$$-2 + 21 = 19$$

$$19 = 19 \quad \checkmark$$

b. $a = -2, b = -5$

c. $u = 8, v = 0$

d. $x = 0, y = -2$

e. $m = 10, n = -2$

f. $s = 0, t = 8$

g. $c = 2, d = -3$

h. $w = 0, x = 5$

i. $x = -4, n = 7$

j. $w = 1, a = -1$

k. $n = 8, y = 6$

l. $c = -152, y = 68$

m. $g = -\frac{1}{3}, h = \frac{13}{6}$

n. $w = \frac{19}{7}, h = \frac{23}{7}$

o. $w = -\frac{2}{3}, m = 1$

p. $a = -\frac{13}{30}, q = \frac{23}{30}$

